Integration, Fundamental Theorem of Calculus

December 2, 2016

Problems

Problem 1. Find the following.

1. $\int_0^{2\pi} \cos(x)dx$

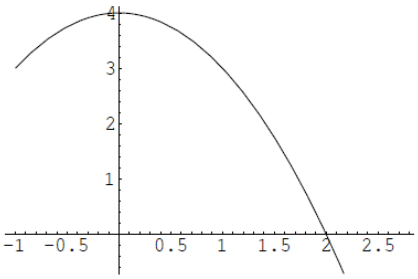
2. The **unsigned** area bounded by $\cos(x)$ between 0 and 2π.

3. $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$

4. $\int_{-1}^{1} t^3(1+t^4)^3 dt$

5. $\int_0^{\pi/4} \tan x dx$

Problem 2. Below is the graph of a function f.



Let $g(x) = \int_0^x f(t)dt$. Find $g(0), g'(0)$ and $g'(2)$.

For $0 < x < 2$ the function $g(x)$ is

1. increasing and concave up;
2. increasing and concave down;
3. decreasing and concave up;
4. decreasing and concave down.

Problem 3. Find the area of the propeller-shaped region enclosed by the curves $x - y^{1/3} = 0$ and $x - y^{1/5} = 0$.

Problem 4. Let $f(x) = \int_{x^2}^{x^3} (t^2 - t)^2 dt$. Find $f'(x)$.

Problem 5. A rocket lifts o the surface of Earth with a constant acceleration of 20 m/sec^2. How fast will the rocket be going 1 minute later?

Problem 6. Compute the integral $\int \sqrt{1-x^2}dx$. (Hint: $u = \arcsin x$ means $x = \sin u$.)

Use it to compute $\int_{-1}^{1} \sqrt{1-x^2}dx$. Does the result match what you would expect from the usual geometric considerations?

Problem 7. Using definite integrals, find the limit of the following sum:

$$\lim_{n\to\infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \right)$$

(Hint: $\dfrac{1}{n+i} = \dfrac{1}{n} \cdot \dfrac{1}{1+\frac{i}{n}}$)

Problem 8. Using **Riemann sums**, find the formula for computing the volume of a cone of height h and radius r. You can use the formula for the volume of a cylinder.